ANALYSIS OF ANNUAL MAXIMUM RAINFALL SERIES IN BENIN CITY USING GENERALIZED EXTREME VALUE (GEV), GENERALIZED LOGISTICS (GLO) AND GENERALIZED PARETO (GPA) DISTRIBUTION BY METHOD OF L-MOMENT

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Abstract
Determination of the magnitude of peak rainfall for various return periods is an essential ingredient for the accurate design of hydraulic structures such as drains, culverts and dams as well as identification of flood risk areas. The aim of this study is to analyze annual rainfall series in Benin City using three parameter probability distribution model, namely; generalized extreme value distribution (GEV), generalized logistics distribution (GLO) and generalized pareto distribution (GPA) with the view of identifying the best fit probability distribution model.

Forty (40) year's annual maximum rainfall series was employed for the analysis. Specific time series analysis test, namely; outlier detection, test of normality and homogeneity were conducted to ensure that the data used are adequate and suitable. Descriptive statistics such as the sample mean, variance, standard deviation, skewness, kurtosis, coefficient of variation were computed using basic statistical equations. The probability weighted moment parameters (b0, b1, b2 and b3), L-Moment values ($\lambda 1$, $\lambda 2$, $\lambda 3$ and $\lambda 4$), L-Moment ratio ($\tau 2$, $\tau 3$ and $\tau 4$) including the distribution parameters, namely; shape (k), scale (α) and location (ξ) parameters were computed based on L-moment procedures.

Results obtained indicates that generalized logistics probability distribution GLO is the best fit distribution model for analyzing the annual maximum rainfall series in Benin City. The predicted rainfall quantile magnitude (Qt) based on the GLO model ranges from 425.877mm at 2years return period to 762.759mm at 200years return period. The coefficient of determination (r^2) for the observed versus predicted rainfall based on the best fit model was observed to be 0.9793. It was thereafter concluded that L-moments and L –moment ratios are useful summary statistics for analyzing annual maximum rainfall data.
Keywords: *L-moments, probability distribution, normality test, goodness of fit statistics, coefficient of variation.*

Introduction

Effective analysis and determination of extreme flood discharge requires the use of statistical frequency analysis or fitting of probability distribution to the series of recorded annual maximum discharge (AMD) (Ehiorobo and Izinyon, 2013; Vivekanandan, 2015; Sharma and Singh, 2010). One of the widely used statistical frequency analysis methods is univariate frequency analysis technique. Univariate frequency analysis is widely used for analyzing hydrological data, including rainfall characteristics, peak discharge series and low flow record of observations. The basic assumption is that the data to be used must be satisfactorily homogeneous otherwise; the estimated probabilities or variable magnitude will be inaccurate (ECOST, 2012). The versatility of statistical frequency analysis makes it the most commonly used procedure for the analysis of precipitation data. A number of probability distribution models such as; Generalized Extreme Value (GEV), Generalized Pareto (GPA), Generalized Logistics (GLO), Gumbel distribution and Normal distribution are used in precipitation frequency analysis (Hosking and Wallis 1997). To employ any type of probability distribution for precipitation frequency analysis, the parameters of the distribution must first be estimated. Different types of probability distribution parameters estimation methods exist, namely; least square regression method (LSR), maximum likelihood estimation (MLE), method of moments (MOM) and method of L-moment. Of the four parameters estimation method, method of moments (MOM) and method of L-moment are widely used owing to their high level of sensitivity to rainfall and runoff data (Ahmad et al., 2011). Method of moments (MOM) has found a wide range of applicability in recent time based on it's used for the determination of parameters of different probability distributions. One of the challenges of MOM is that for small sample size, the numerical values of sample moments can be very different from those of the probability distribution from which the sample was drawn (Landwehr, 1979, Ehiorobo and Izinyon, 2013). On account of these, alternative approach such as L-moments (LMO) was introduced to accurately estimate the parameters of probability distributions. L-Moment is a dramatic improvement over conventional product moment statistics for characterizing the shape of a probability distribution and estimating the distribution parameters, particularly for environmental data where sample sizes are commonly very small (Hosking, 1990; Izinyon and Ehiorobo, 2015).

L-Moment Theory and Statistics

L-moments can be obtained by considering linear combinations of the observation in a sample of data that has been arranged in ascending order. The basic steps in the determination of L-Moment statistics as presented in Hosking and Wallis, (1997) and Eregno, (2014). are described as follows;

Computation of probability weighted moments of distribution (pwms)

Probability weighted moments is needed for the calculation of L-moment. The data must first be ranked in ascending order of magnitude, thereafter; the following equations proposed by cunnane, 1989 can thus be applied

$$b_0 = \frac{1}{N}\sum_{j=1}^{n} X_{(j:n)} \quad (1)$$

$$b_1 = \frac{1}{N}\sum_{j=2}^{n} X_{(j:\mathrm{n})}\,[(j-1)/(n-1)] \quad (2)$$

$$b_2 = \frac{1}{N}\sum_{j=3}^{n} X_{(j:\mathrm{n})}\,[(\mathrm{j}-1)(\mathrm{j}-2)]/[(\mathrm{n}-1)(\mathrm{n}-2)] \quad (3)$$

$$b_3 = \frac{1}{N}\sum_{j=4}^{n} X_{(j:\mathrm{n})}\,[(\mathrm{j}-1)(\mathrm{j}-2)(\mathrm{j}-3)]/[(\mathrm{n}-1)(\mathrm{n}-2)(\mathrm{n}-3)] \quad (4)$$

Where;

$X_{(j)}$ represent the ranked annual maximum series in which $X_{(1)}$ is the smallest precipitation or stream flow data and $X_{(n)}$ is the largest. The parameters (b_0, b_1, b_2 and b_3) can easily be determined by using the developed Microsoft excel algorithm.

Computation of L-Moment Values

L-moment values are easily calculated in terms of probability weighted moment (PWMs). In particular, the first four L-moment values are given as follows (Hosking and Wallis, 1997).

$$\lambda_1 = L_1 = b_0 \quad (5)$$

$$\lambda_2 = L_2 = (2b_1 - b_0) \quad (6)$$

$$\lambda_3 = L_3 = (6b_2 - 6b_1 + b_0) \quad (7)$$

$$\lambda_4 = L_4 = (20b_3 - 30b_2 + 12b_1 - b_0) \quad (8)$$

The parameters ($\lambda_1 = L_1$; $\lambda_2 = L_2$; $\lambda_4 = L_4$) can easily be determined by using the developed Microsoft excel algorithm that requires forty year's annual maximum monthly rainfall or discharge data.

Computation of L-Moment Ratio

L-Moment ratio used for expressing the parameter estimates are as follows (Hosking and Wallis, 1997).

$$L - CV \text{ (Coefficient of variability)} = (\tau_2) \quad (9)$$

$$L - \text{Skewness} = (\tau_3) \quad (10)$$

$$L \text{ - Kurtosis} = (\tau_4) \quad (11)$$

L-Cv is a dimensionless measure of variability. For a distribution or sample data that only has positive values, L-Cv is normally in the range of 0 < |L-Cv| < 1. Negative values of L-Cv are only possible if the at-site mean has a negative value (Sanjib, 2016; Herlina, 2015). L-Skewness is a dimensionless measure of asymmetry, which may take on positive or negative values. For a distribution or sample data, L-skewness is in the range 0 < |L-Skewness| < 1 (CEH, 2001). L-kurtosis refers to any measure of the "peakedness" of the probability distribution of a real-valued random variable. The parameters (τ_2 τ_3 and τ_4) are computed using the formula presented in Hosking and Wallis, (1997), Gubareva and Gartsman, (2010).

$$\tau_2 = \frac{\lambda_2}{\lambda_1} = \frac{L_2}{L_1} \quad (12)$$

$$\tau_3 = \frac{\lambda_3}{\lambda_2} = \frac{L_3}{L_2} \quad (13)$$

$$\tau_4 = \frac{\lambda_4}{\lambda_3} = \frac{L_4}{L_3} \quad (14)$$

The parameters (τ_2 τ_3 and τ_4) can easily be determined by using the developed Microsoft excel algorithm that requires fourty year's annual maximum monthly rainfall or discharge data.

Advantages of L-Moment

The main advantage of L-moment over conventional moments is that L-moments, being linear functions of the data, suffer less from the effects of sampling variability and L-moments are more robust compared to conventional moments in handling outliers in the data. In addition, L-moment enables more secure inferences to be made from small samples about an underlying probability distribution (Hosking & Wallis, 1997). Some of the underlying simplicity of L-Moments are;

i. L-moment is based on linear combination of data that have been arranged in ascending order of magnitude. It provides an advantage as it is easier to work with, and more reliable since it is less sensitive to outliers.
ii. The method of L-moment calculates more accurate parameter than method of moment (MOM) technique especially for smaller sample size.
iii. MOM techniques only apply to limited range of parameters, whereas L-moment can be more widely used, and are also nearly unbiased

Research Methodology

Description of study area

Edo State was created from the defunct Bendel State on the 21 August, 1991. The State is located in the rain forest belt of Nigeria between longitude 5° E and 6° 42"E and latitude 5° 45" N and 35" N. It is bounded by Kogi State to the North; to the East both Kogi and Anambra States; to the South by Delta State and by Ondo State to the West. The state is generally low-lying except in the Northern part that is characterized by undulating hills. The State experiences both wet and dry seasons. The wet season lasts from April to November while, the dry season lasts from December to March.

Figure 1: The Map of Edo State of Nigeria (Ref: Edo State Government, 2007)

Data collection

The data used for this study was collected from the Nigerian Meteorological Agency, Oshodi; Lagos State, Nigeria. The data includes monthly precipitation data for 40 years spanning between; 1974 to 2013. The data were then analyze using Microsoft excel program to obtain the annual maximum precipitation records.

Date Analysis

Preliminary analysis of the data was aimed at; testing the assumption of same population distribution, testing the assumption of normality and assessment of seasonal variability. Frequency analysis of data requires that the data be homogeneous and independent. Homogeneity test was carried out to establish the fact that the data used for the analysis are from the same population distribution. Homogeneity test is based on the cumulative deviation from the mean as expressed using the mathematical equation presented in Raes et al., (2006).

$$S_k = \sum_{i=1}^{k} \left(X_i - \bar{X} \right) \quad k = 1, \text{----------} \; n \qquad (15)$$

where

= The record for the series X_1 X_2 ----------- X_n

= The mean; S_{ks} = the residual mass curve

For a homogeneous record, one may expect that the S_{ks} fluctuate around zero in the residual mass curve since there is no systematic pattern in the deviation of X_i. To perform the homogeneity test, a software package (Rainbow) for analyzing hydrological data was employed (Raes et al., 2006).

The stochastic nature of meteorological data makes it difficult for them to obey the assumptions of normality. For normality, the histogram and Q-Q plot should visually indicate that the data are approximately normally distributed. The histogram plot should assume the characteristic bell shape configuration. The seasonal variability test was aimed at visualizing the fluctuating effects of climatic variables occasioned by change in time and season. The time series plot in Microsoft Excel program was employed to visualize the presence of seasonal variation in the data.

The underlying equations of the selected probability distribution models employed for this study, namely; Generalized Extreme Value (GEV), Generalized Pareto (GPA) and Generalized Logistics (GLO) are presented in (Table 1). To compute the parameters of the distributions including The probability weighted moment parameters (b0, b1, b2 and b3), L-Moment values (ƛ1, ƛ2, ƛ3 and ƛ4), L-Moment ratio (Ʈ2, Ʈ3 and Ʈ4), a Microsoft Excel algorithm was developed based on the mathematics of L-moment. To select the probability distribution model that best fit the annual maximum rainfall data, five selected goodness of fit statistics were employed as presented in (Table 2). The selected goodness of fit statistics includes; root mean square error (RMSE), relative root mean square error (RRMSE), maximum absolute deviation index (MADI), maximum absolute error (MAE) and probability plot correlation coefficient (PPCC). The goodness of fit statistics (RMSE, RRMSE, MADI, MAE and PPCC) were selected since they can adequately assess the fitted distribution at a site. They also possess an added advantage of being able to summarize the deviation between observed precipitation and predicted precipitation.

Table 1: Basic Equations of Selected Probability Distribution Models

Distribution	Parameters/ Range	Probability density function f(x)	Cumulative distribution function F(x)	Quantile function (x_p)
Generalized Extreme Value (GEV)	Parameters: ξ (location), α (scale), k(shape) Range: α > 0, ξ + α/k ≤ x < ∞ for k < 0, -∞ ≤ x ≤ ξ + α/k for k > 0	$f(x)=\frac{1}{\alpha}\left[1-\frac{k(x-\xi)}{\alpha}\right]^{1/k-1}\exp\left\langle-\left[1-\frac{k(x-\xi)}{\alpha}\right]^{1/k}\right\rangle$	$F(x)=\exp\left(-\left[1-\frac{k(x-\xi)}{\alpha}\right]^{1/k}\right)$	$x_p=\xi+\frac{\alpha}{k}(1-[-\ln(F)]^k)$
Generalized Pareto (GPA)	Parameters: ξ (location), α (scale), k(shape) Range: α > 0, ξ ≤ x < ∞ for k < 0, ξ ≤ x ≤ ξ + α/k for k > 0	$f(x)=\frac{1}{\alpha}\left[1-k\frac{x-\xi}{\alpha}\right]^{1/k-1}$	$F(x)=1-\left[1-k\frac{x-\xi}{\alpha}\right]^{1/k}$	$x_p=\xi+\frac{\alpha}{k}\left[1-(1-F)^k\right]$
Generalized Logistics (GLO)	Parameters: ξ (location), α (scale), k(shape) Range: α > 0, ξ + α/k ≤ x < ∞ for k < 0, -∞ ≤ x ≤ ξ + α/k for k > 0	$\gamma=\left[1-\frac{k(x-\xi}{\alpha}\right]^{\frac{1}{K}}$ for k ≠ 0 $f_x=\left(\frac{1}{\alpha}\right)\left[\frac{\gamma^{(1-x)}}{(1+\gamma)}\right]^2$	$F_x(x)=\frac{1}{1+\gamma}$	$x_p=\xi+\frac{\alpha}{k}\left[1-\left(\frac{1-F}{F}\right)^k\right]$

Table 2: Selected goodness of fit statistics

Statistics	Equation
Root mean square error (RMSE)	$RMSE = \left(\frac{\sum (x_i - y_i)^2}{n-m} \right)^{½}$
Relative root mean square error (RRMSE)	$RRMSE = \left(\frac{\sum \left(\frac{x_i - y_i}{x_i} \right)^2}{n-m} \right)^{½}$
Mean absolute deviation index	$MADI = \frac{1}{N} \sum_{i=1}^{N} \left\| \frac{x_i - y_i}{x_i} \right\|$
Maximum Absolute Error	$MAE = \max (\|x_i - y_i\|)$
Probability Plot Correlation Coefficient	$PPCC = \frac{\sum [(x_i - \bar{x})(y_i - \bar{y})]}{\left[\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2 \right]^{1/2}}$

Results and Discussion

To test the normality assumption of the data used in this study, a column plots presented in Figure 2 was done using Microsoft excel program

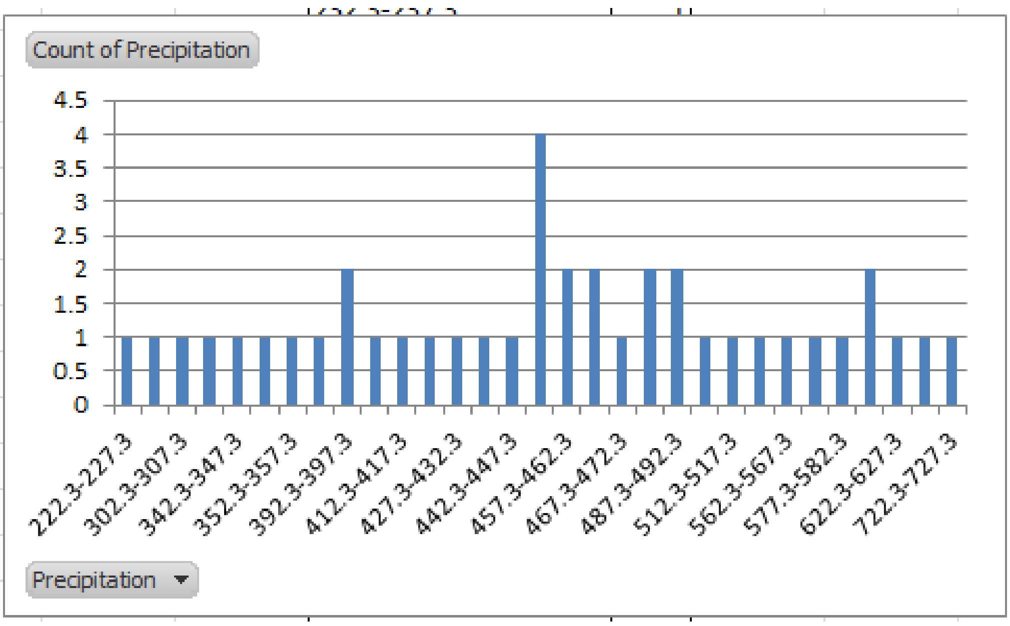

Figure 2: Normality test of precipitation data

From the result of Figure 2, it was observed that the shape of the plot did not produce the characteristic bell shape configuration reminiscence of a normal probability distribution curve. Hence, it was concluded that the precipitation data is not normally distributed which is expected owing to the stochastic nature of most climatic variables such as rainfall. In addition, the cumulative density plot presented in Figure 3 shows the non-linear nature of the data used which further support the fact that the rainfall data did not obey normality.

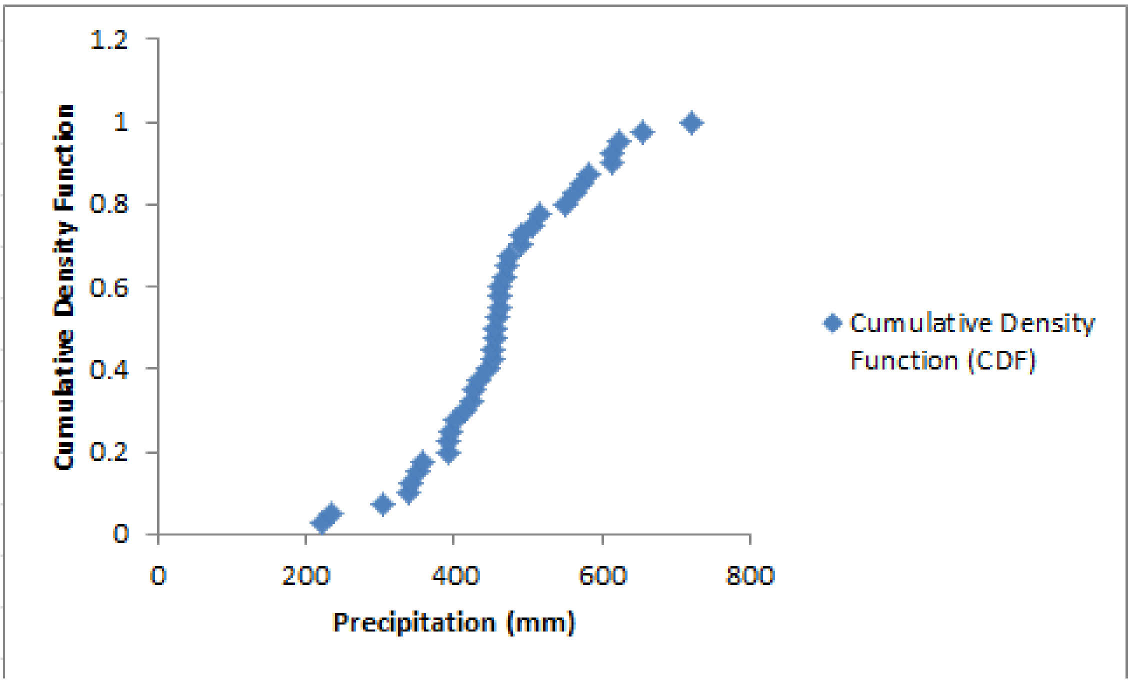

Figure 3: Cumulative density function against observed precipitation

The time series plot presented in Figure 4.3 shows the presence of seasonal variability since rainfall depth varies within the period understudy as some years experienced extreme precipitation compared to others.

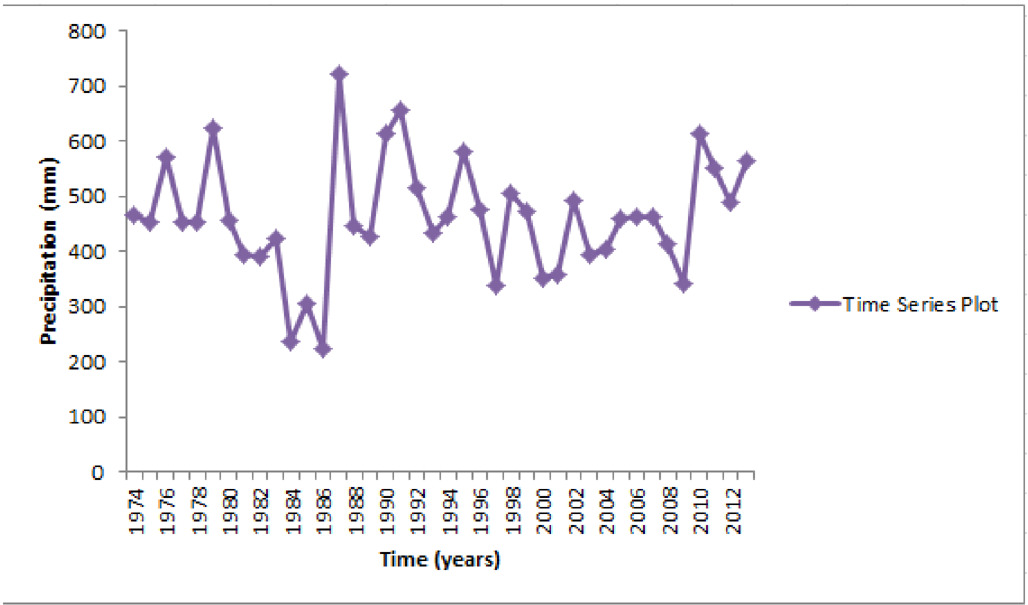

Figure 4: Time series plot

On whether the rainfall data used in this study are from the same population distribution, homogeneity test was performed using hydrological software (RAINBOW). Result of the test presented in Figure 5 shows that the data used is homogeneous.

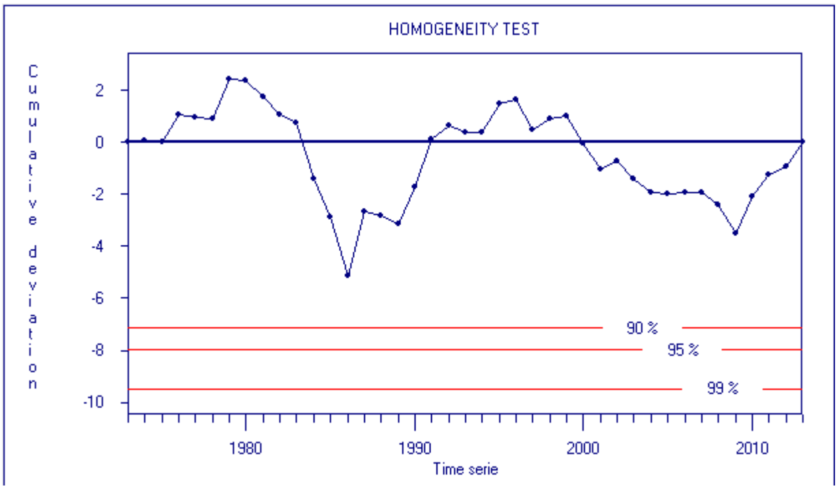

Figure 5: Homogeneity test of data

For a homogeneous record, the rainfall data points normally fluctuate around the zero center line in the residual mass curve as observed in Figure 5. The descriptive statistics and L-moment statistics which includes; the probability weighted moment statistics (b0, b1, b2 and b3), computed L-moment values (ƛ1, ƛ2, ƛ3 and ƛ4) and the L-moment ratio values are presented in Table 2

Table 2: Descriptive and L-moment Statistics

Probability Weighted Moment Values	L-Moment Values	L-Moment Ratios	Basic Statistics
b_0 = 461.565	λ_1 = 461.565	L-CV = 0.128	n = 40
b_1 = 260.3925	λ_2 = 59.220	L-Skewness = 0.0347	Mean = 461.565
b_2 = 183.807	λ_3 = 2.0527	L-Kurtosis = 0.2151	Variance = 11280.1
b_3 = 143.190	λ_4 = 12.739		Standard Dev. = 106.208
			C.V = 0.230

The L-moments parameters λ_1 and λ_2, their ratio ($\tau=\frac{\lambda_2}{\lambda_1}$) termed L-CV, and L-moment ratios λ_3 and λ_4 are the most useful quantities for summarizing probability distribution (Maleki-Nezhad, 2006). The value of λ_1 (L- mean) is a measure of central tendency; λ_2 (L- standard deviation) is a measure of dispersion and L-CV (λ) is the coefficient of L-variation. L-Skewness (λ_3) measures whether the distribution is symmetric with respect to the dispersion from the mean and L-kurtosis (λ_4) refers to the weight of the tail of a distribution. The values of λ_3 and λ_4 are constrained to be between -1 and +1 and λ_4 is constrained by λ_3 to be no lower than -0.25 (Eslamian and Feizi, (2007). The parameters of location (ξ), scale (α) and shape (k) of the selected probability distribution models is presented in Table 3

Table 3: Probability distribution parameter estimate based on L-moment

Distribution Model	Shape (k)	Scale (α)	Location (ξ)
GEV	0.223348438	74.54719112	540.6930545
GLO	-0.03466283	57.98293611	425.8765149
GPA	0.865993717	316.7041936	291.8408521

The estimated parameters for the different distributions were applied to the relevant quantile function given in Table 1. The predicted annual maximum precipitation records based on L-moment using the three probability distribution model, namely; GEV, GLO and GPA was obtained and the best from among candidate distributions fitted to the observed data at the station was selected by subjecting their respective predicted values to five statistical goodness-of-fit tests. The computed goodness of fit statistics based on the three probability distribution models, namely; GEV, GLO and GPA is presented in Table 4

Table 4: Computed goodness of fit statistics

GoF Statistics	RMSE	RRMSE	MADI	MAE	PPCC
GEV	115.6429728	0.326199623	0.269482909	0.45673	0.9675
GLO	37.79785402	0.079475417	0.001362853	0.00354	0.9793
GPA	30.20182487	0.089637382	0.006644185	0.0674	0.9171

The overall goodness of fit of each distribution was judged using a ranking scheme by comparing the three categories of test criteria based on the relative magnitude of the statistical test results. The distribution with the lowest RMSE, lowest RRMSE, lowest MADI, lowest MAE and highest PPCC was assigned a score of 3, the next was given the score 2, while the worst was given the score 1. The overall score of each distribution was obtained by summing the individual point scores and the distribution with the highest total point score was selected as the best fit distribution model. The scoring scheme and the overall ranking of the distributions models at the stations based on the goodness of fit tests is presented in Table 5

Table 5: Scoring and ranking scheme for selected probability distribution models

Test Criteria	Distribution Scoring		
	GEV	GLO	GPA
RMSE	1	2	3
RRMSE	1	3	2
MADI	1	3	2
MAE	1	3	2
PPCC	2	3	1
Total Score/ Rank	6 (3rd)	14 (1st)	10 (2nd)

Based on the result of Table 4.4, generalized logistics probability distribution (GLO) with the highest total score of 14 was selected as the best probability distribution model for analyzing annual maximum rainfall series in Benin City followed by GPA and then GEV. The quantile estimates (Qt) based on 2, 5, 10, 20, 50, 100, 200 and 500 years was also obtained based on L-moment procedure using the best fit probability distribution model (GLO). Results is presented in Table 6.

Table 6: Computed quantile estimates based on selected return periods using GLO

Z	AA	AB	AC	AD	AE	AF
Return Periods (T)	(T - 1)	[(T - 1)^-K]	R = [1 -(T - 1)^-K]	Z = [(α/k)*(R)]	QT = (ξ + Z)	Exceedence Probability E = (1/T)
2	1	1	0	0	425.8765149	0.5
5	4	1.049226143	-0.049226143	82.34400664	508.2205215	0.2
10	9	1.079137404	-0.079137404	132.3786615	558.2551764	0.1
20	19	1.107452783	-0.107452783	179.7437724	605.6202873	0.05
50	49	1.144424059	-0.144424059	241.5882084	667.4647233	0.02
100	99	1.172666081	-0.172666081	288.8306102	714.7071251	0.01
200	199	1.201392049	-0.201392049	336.8825427	762.7590576	0.005
500	499	1.240291595	-0.240291595	401.952529	827.8290439	0.002
1000	999	1.27049637	-0.27049637	452.478166	878.3546809	0.001

The graphical visualization of the observed and predicted annual maximum precipitation based on the three probability distribution models, namely; GEV, GLO and GPA was also obtained and presented in Figures 5, 6 and 7 respectively.

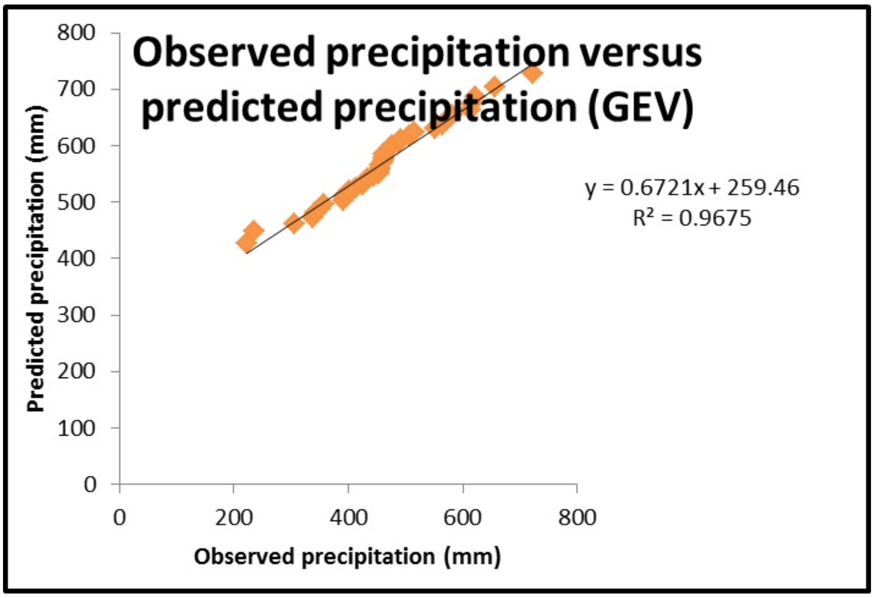

Figure 5: Observed versus predicted precipitation based on GEV distribution

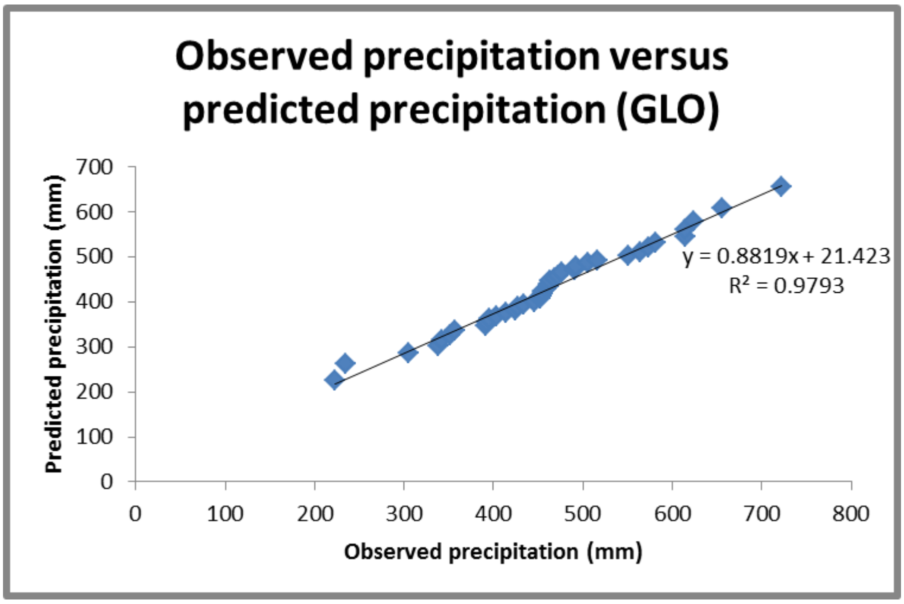

Figure 6: Observed versus predicted precipitation based on GLO distribution

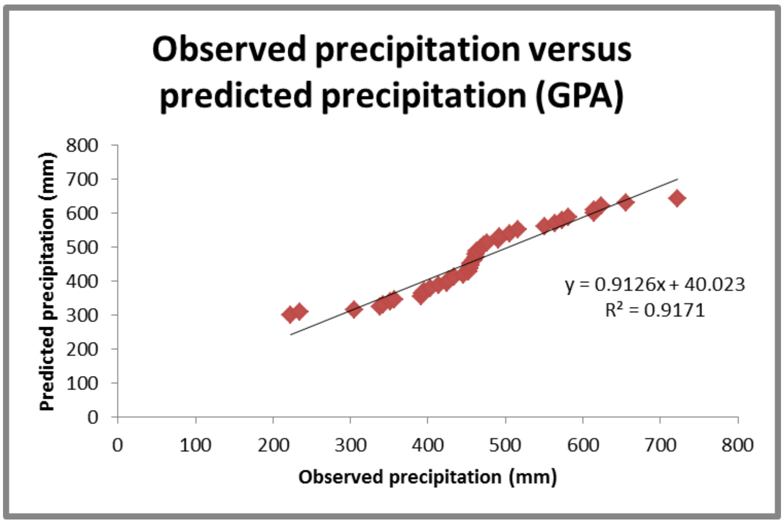

Figure 7: Observed versus predicted precipitation based on GPA distribution

The computed coefficient of determination (r^2) between the observed and predicted precipitation was observed to 0.9675 for generalized extreme value distribution (GEV), 0.9793 for generalized logistics distribution (GLO) and 0.9171 for generalized Pareto (GPA) distribution. Based on the computed coefficient of determination, it was concluded that generalized logistics distribution had a better fit of the annual maximum precipitation data from Benin City.

Conclusion

The paper gave a detail description of the current method of statistical parameter estimation of selected probability distribution models, namely; Generalized Extreme Value (GEV), Generalized Logistics (GLO) and Generalized Pareto (GPA) probability distributions using L-moment method. A simple to use Microsoft Excel Algorithm have been developed for estimating basic descriptive statistics such as the sample mean, variance, standard deviation, skewness, kurtosis, and coefficient of variation. Other exciting features include; the computation of Probability weighted moment parameters (b0, b1, b2 and b3), L-Moment values (ƛ1, ƛ2, ƛ3 and ƛ4), L-Moment ratio values (Ʈ2, Ʈ3 and Ʈ4), and goodness of fit statistics (RRMSE, RMSE, MADI, MADI and PPCC). The algorithm will not only find use in the practice of engineering hydrological computation, it will also help design engineers in estimating the magnitude of peak rainfall/discharge for various return periods.

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