APPLICATION OF TOPSIS BASED ON ENTROPY WEIGHTS FOR SELECTION OF SUPPLIER PROBLEM

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Abstract
In this paper, the problem of selecting the best supplier from four alternatives was considered. Entropy method was used to determine the weights of the four criteria involved by ranking the alternatives under each criterion. The entropy and degree of diversification of each criterion was obtained from which the criteria weights were obtained. Technique for Order Preference based on Similarity to Ideal Solution (TOPSIS) was then applied to the resulting decision matrix to determine the best alternative based on all the criteria. In order to ensure the robustness of the decision obtained, sensitivity analysis was carried out by perturbing some of the weights of criteria. The result of the analysis shows that irrespective of the criteria weights the same decision was upheld using the present method. The utility of the technique has been demonstrated for solving supplier selection problem.

Keywords: TOPSIS, entropy, decision matrix, weight, sensitivity analysis, pairwise comparison

INTRODUCTION

Decision making is undoubtedly one of the most important activities which managers carry out in any business. It is one of the most significant and omnipresent human activity (Garg et al, 2015). The decision making process is complicated by a multiplicity of alternatives which must be compared under different criteria which may be conflicting. Consequently, a tradeoff between the relevant criteria is essential in selecting the right decision or alternative (Tahriri et al, 2008, Kharchenko et al, 2014). The interest in the supplier selection problem has received enormous attention from both academicians and practitioners as a result of the benefits which accrue from good buyer supplier relationship. Increase in the quality of products and a reduction in inventory related costs are a few benefits highlighted by Chamodrakas et al (2010)

The multi criteria decision making (MCDM) problem consist of making a choice among a finite set of alternatives with respect to multiple criteria (Choo et al, 1999). A number of quantitative techniques (Sreekumar and Mahapatra, 2009) techniques are available for addressing the multi criteria decision making problem. They include but not limited to Analytic Hierarchy process (AHP), Technique for Order Preference by Similarity to ideal solution (TOPSIS) and Data Envelopment Analysis (DEA) Analytic Hierarchy process addresses the MCDM by breaking it down into a hierarchy of decision criteria and decision alternatives.(Ozkan et al, 2011). The Analytic Hierarchy process compares criteria or alternatives with respect to criterion in a natural pairwise mode (Kumar et al, 2009). TOPSIS is a goal based approach for finding the alternative that is closest to the ideal solution (Bhutia and Phipon, 2012). TOPSIS was developed by Hwang and Yoon (1981). The determination of criteria weights is a very important step in multi criteria decision making process.

A number of researchers have used different methods for the supplier selection problem. Wu and Liu (2013) used entropy method with VOKOR algorithm and fuzzy TOPSIS with vague set method. Deng et al (2014) applied AHP based on a new and feasible representation of uncertain information called D numbers to the supplier selection problem. Mohaghar et al (2013) used extended TOPSIS technique with interval valued triangular fuzzy numbers for ranking contractors. Green supplier selection was carried out using a compromised criteria weight generation technique and TOPSIS by Freeman and Chen (2015). AHP and TOPSIS methods were applied by Onder and Dag (2013) to a supplier selection problem using a cable manufacturing company as a case study. In this paper, a supplier selection problem was considered using TOPSIS methodology. The criteria weights which are a very important consideration were obtained using the Shannon entropy method. The entropy method determination of criteria weights is more effective as it does not result in inconsistency issues as is the case with AHP method for criteria weight determination. The robustness of the decision reached was subjected to sensitivity analysis.

ENTROPY

Entropy method is used to determine the weights of coefficients used for multi criteria decision making. Entropy is a measure of disorder degree of a system (Lui and Cui, 2008). The entropy method (Lin and Wen, 2009) is an object empowerment approach, in which the weight values of individual indicators are determined by calculating the entropy and entropy weight (Lee et al, 2011)

The entropy method is explained using the following steps. Using the ranking for the different alternatives under the respective criteria represented by the matrix below

Step 1: Normalize the decision matrix.

$$set\ p_{ij} = \frac{x_{ij}}{\sum_{j=1}^{M} x_{ij.}},\ j = 1,\dots\dots\dots m\ ,\ i = 1,\dots\dots.n \qquad (1)$$

Where m denotes the number of criteria and n denotes the number of alternatives. In equation (1) x_{ij} is the rank according to table 1 of alternative i under criterion j.

Step 2: Compute the entropy H_i

$$Hi = -h_0 \sum_{j=1}^{m} p_{ij} \ln p_{ij} \quad i=1,\dots..n \qquad (2)$$

where h_o is the entropy constant and $h_o = -\frac{1}{\ln m}$.

Step 3: Set $d_i = 1 - h_1$, i =1,………,n as the degree of diversification.

Step 4: Obtain $\sum H_i$ (3)

Step 5: Obtain $d_s = m - \sum H_i$ (4)

Step 6: $w_i = \frac{d_i}{\sum_{s=1}^{n} d_s} \qquad i = 1, \dots n$ (5)

TOPSIS

TOPSIS finds the best alternatives by minimizing the distance to the positive ideal solution and maximizing the distance to the negative ideal solution.

All alternative solutions can be ranked according to their closeness coefficient to the ideal solution.

TOPSIS METHOD

Step 1: Calculate the normalized decision matrix A. The normalized value r_{ij} is calculated as:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \qquad (1 \le i \le m, 1 \le j \le n) \qquad (6)$$

Step 2: Calculate the weighted normalized decision matrix:

$$v_{ij} = (r_{ij} w_{ij}) \qquad (7)$$

Where w_j is the weight of the j^{th} criterion and $\sum_{j=1}^{n} w_j = 1$

Step 3: Calculate the positive ideal solution (v^{+}) and the negative ideal solution (v^{-})

$$A^{+} = \{v_1{}^{+}, v_2{}^{+}, .. v_n{}^{+}\} = \{(\max v_{ij} \mid j \in J), (\min v_{ij} \mid j \in N) \mid i = 1,2, \dots m\} \quad (8)$$

$$A^{-} = \{v_1{}^{-}, v_2{}^{-}, .. v_n{}^{-}\} = \{(\min v_{ij} \mid j \in J), (\max v_{ij} \mid j \in J) \mid i = 1,2, \dots m\} \quad (9)$$

Where $J = \{j = 1,2, \dots, n \mid j \ associated\ with\ benefit\ criteria\}$

$N = \{j = 1,2, \dots, n \mid j \ associated\ with\ cost\ criteria\}$

Step 4: Calculate the separation measures:

$$S^{+} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v^{+})^2}, \ where\ (1 \le i \le m, 1 \le j \le n) \quad (10)$$

$$S^{-} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v^{-})^2}, \ where\ (1 \le i \le m, 1 \le j \le n) \quad (11)$$

Step 5: Calculate the relative closeness coefficient to the ideal solution:

$$CC_i{}^{*} = \frac{S_i{}^{-}}{S_i{}^{+} + S_i{}^{-}} \quad (1 \le i \le m, 1 \le j \le n) \quad (12)$$

Step 6: Rank the preference order. A set of alternative can now be preference ranked according to the descending order of $CC_i{}^{*}$

NUMERICAL EXAMPLE

A manufacturer has a problem of selecting a suitable vendor for the supply of some equipment. A large number of contractors submitted their bids for the pre-qualification exercise. Four contractors have passed through the pre-qualification stage. One contractor has to be selected based on the four factors. To determine the weights of the criteria, the decision makers first rate the alternatives under the four criteria using the following grading system proposed by Ghorbani et al (2012)

Table 1: Grading system for weight determination

Grading	Status
7	Definitely satisfactory
6	Almost satisfactory
5	Satisfactory
4	Average
3	Unsatisfactory
2	Almost unsatisfactory

The grading system shown in Table 1 was applied by a group of experts to rank the alternatives under the four criteria and the data shown in Table 2 was obtained

Table 2: Ranking of four alternatives under the four criteria

	A_1	A_2	A_3	A_5
C_1	5	3	3	3
C_2	5	3	1	7
C_3	3	4	3	3
C_4	3	3	2	6

We begin by finding the weights of the four criteria using the entropy method. Using equation (1) we obtain the normalized matrix from Table 2 and from equations 2-5 we obtain the entropy, degree of diversification and the weights of the criteria. It can be observed that the sum of the criteria weights equal unity.

Table 3: Entropy, degree of diversification and weights of criteria

	C_1	C_2	C_3	C_4
H_i	0.9772	0.9939	0.9455	0,9484
d_i	0.0228	0.0061	0.0548	0.0516
W_i	0.1688	0.0453	0.4036	0.3823

The pairwise comparison matrices for the suppliers with respect to each criterion are presented in Tables 4 to 7

Table 4: Pairwise comparison matrix with respect to criteria C_1

C_1	S_1	S_2	S_3	S_4
S_1	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
S_2	5	1	2	2
S_3	3	$\frac{1}{2}$	1	$\frac{1}{3}$
S_4	6	$\frac{1}{2}$	3	1

Table 5: Pairwise comparison matrix with respect to criteria C_2

C_2	S_1	S_2	S_3	S_4
S_1	1	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{7}$
S_2	6	1	2	$\frac{1}{2}$
S_3	3	$\frac{1}{2}$	1	$\frac{1}{3}$
S_4	7	2	3	1

Table 6: Pairwise comparison matrix with respect to criteria C_3

C_3	S_1	S_2	S_3	S_4
S_1	1	4	5	3
S_2	$\frac{1}{4}$	1	2	$\frac{1}{7}$
S_3	$\frac{1}{5}$	$\frac{1}{2}$	1	$\frac{1}{5}$
S_4	$\frac{1}{3}$	7	5	1

Table 7: Pairwise comparison matrix with respect to criteria C_4

C_4	S_1	S_2	S_3	S_4
S_1	1	6	$\frac{1}{3}$	2
S_2	$\frac{1}{6}$	1	$\frac{1}{2}$	$\frac{1}{3}$
S_3	3	5	1	4
S_4	$\frac{1}{2}$	4	$\frac{1}{4}$	1

We begin by finding the weights of the four criteria using the entropy method. Using equation (1) we obtain the normalized matrix from Table 8 and from equations 2-5 we obtain the entropy, degree of diversification and the weights of the criteria. It can be observed that the sum of the criteria weights equal unity.

Table 8: Entropy, degree of diversification and weights of criteria

	C_1	C_2	C_3	C_4
H_i	0.9772	0.9939	0.9455	0,9484
d_i	0.0228	0.0061	0.0548	0.0516
W_i	0.1688	0.0453	0.4036	0.3823

The pairwise comparison matrices for the suppliers with respect to each criterion are presented in Tables 9 to 10

Table 9: Pairwise comparison matrix with respect to criteria C_1

C_1	S_1	S_2	S_3	S_4
S_1	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
S_2	5	1	2	2
S_3	3	$\frac{1}{2}$	1	$\frac{1}{3}$
S_4	6	$\frac{1}{2}$	3	1

Table 10: Pairwise comparison matrix with respect to criteria C_2

C_2	S_1	S_2	S_3	S_4
S_1	1	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{7}$
S_2	6	1	2	$\frac{1}{2}$
S_3	3	$\frac{1}{2}$	1	$\frac{1}{3}$
S_4	7	2	3	1

To obtain the decision matrix which will be used for TOPSIS procedure to determine the best alternative, the priority vector was determined for each pairwise comparison matrix with respect to each criterion. The normalized matrices and priority vectors for the data in Tables 4 to 7 are presented in Tables 11 to 14

Table 11: Normalized matrix and priority vector of pairwise comparison matrix with respect to Criterion C_1

	S_1	S_2	S_3	S_4	Priority Vector
S_1	0.0667	0.2000	0.0526	0.0476	0.0917
S_2	0.3333	0.4000	0.3158	0.5714	0.4051
S_3	0.2000	0.2000	0.1579	0.0952	0.1633
S_4	0.4000	0.2000	0.4737	0.2858	0.3399

Table 12: Normalized matrix and priority vector of pairwise comparison matrix with respect to Criterion C_2

	S_1	S_2	S_3	S_4	Priority Vector
S_1	0.0589	0.0655	0.0526	0.0723	0.0573
S_2	0.3529	0.2727	0.3158	0.2530	0.2986
S_3	0.1765	0.1364	0.1579	0.1687	0.1599
S_4	0.4118	0.5455	0.4737	0.5060	0.4842

Table 13: Normalized matrix and priority vector of pairwise comparison matrix with respect to Criterion C_3

	S_1	S_2	S_3	S_4	Priority Vector
S_1	0.5608	0.32	0.3846	0.6908	0.4890
S_2	0.1402	0.080	0.1539	0.0329	0.1017
S_3	0.1122	0.0400	0.0769	0.0461	0.0688
S_4	0.1869	0.5600	0.3846	0.2303	0.3405

The decision matrix can now be formed by using Tables 7 to 10 and the criteria weights in Table 3. The decision matrix is presented in Table 14.

Table 14: Decision matrix

	C_1	C_2	C_3	C_4
Alternatives/Criteria Weights	0.1688	0.0453	0.4036	0.3823
S_1	0.0917	0.0573	0.4890	0.2622
S_2	0.4051	0.2986	0.1017	0.0640
S_3	0.1633	0.1599	0.0688	0.5184
S_4	0.3399	0.4842	0.3405	0.1584

TOPSIS procedure listed in equations 6-12 can now be applied to the decision matrix in Table 11. The weighted normalized matrix was obtained using equation 7 and is shown in Table 12.

Table 15: Weighted normalized decision matrix

S_1	0.0276	0.0044	0.3244	0.1663
S_2	0.1219	0.0228	0.0675	0.0406
S_3	0.0491	0.0122	0.0456	0.3267
S_4	0.1023	0.0370	0.2259	0.1004

The positive ideal (v^+) and the negative ideal solutions (v^-) obtained from Table 15 are presented in Table 16

Table 16: Positive and negative ideal solutions

V^+	0.0491	0.0370	0.3244	0.3267
V^-	0.0276	0.0044	0.0456	0.0406

The separation measures and relative closeness coefficients of the alternatives are shown in Table 17.

Table 17: Relative closeness coefficients

	A_1	A_2	A_3	A_5
s_i^+	0.1652	0.3917	0.2799	0.2525
s_i^-	0.3058	0.0986	0.2871	0.2067
cc_i^+	0.6493	0.2010	0.5064	0.4501

From the computations of the relative closeness coefficients shown in Table 14, it can be deduced that the best alternative is alternative 1 since it has the highest relative closeness coefficient. Sensitivity analysis was carried out and the results are presented in Table 1

DISCUSSION

TOPSIS and entropy methods have been applied to the supplier selection problem. The result of the analysis shows that the four suppliers have been ranked differently when each criterion was considered in isolation. Specifically, under criterion C_1, computations showed that the best supplier was S_2 followed by S_4, S_3 and S_1 in that order. For criterion C_2, the best alternative was S_4 followed by S_2, S_3 and S_1 in that order. For criterion C_3, the alternatives were ranked in the following order S_1, S_4, S_2 and S_3. For criterion C_4, the alternatives were ranked in the following order S_3, S_1, S_4 and S_2. However, when the four criteria were considered together, analysis showed that supplier S_1 emerged as the best alternative as shown by Table 14 since it has the highest closeness coefficient. Further analysis was carried out using sensitivity analysis to investigate the effect of a change in the weights on the final decision.

The weights were perturbed according to the procedure suggested by Memariani et al (2009) and observe the effect on the choice of the best alternative. Table 15 shows the rankings of the alternatives when the weights are perturbed. The weights used for the sensitivity analysis are shown on the x axis. In the first case, the weight of C_1 was increased by 0.15 from the base value resulting in the following weight for the four criteria (0.3188, 0.0371, 0.3308, 0.3133) according to the method proposed by Memariani et al (2009). In the second case, the weight of C_2 was increased by 0.2 resulting in the following weights for the four criteria (0.1122, 0.0301, 0.6036, 0.2541). In both cases, it can be observed that the decision is consistent with the initial decision. The most preferred alternative is still alternative 1 as shown in Figure 1

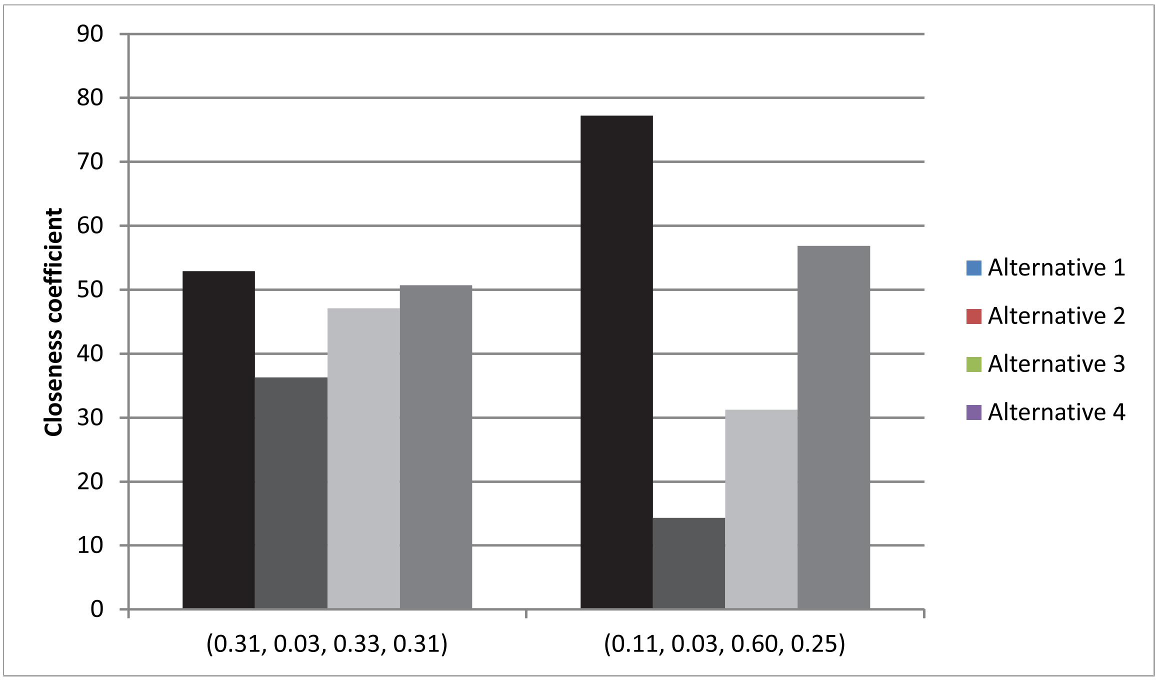

Figure 1: Result of sensitivity analysis for different criteria weights

CONCLUSION

TOPSIS has been applied to a hypothetical problem of supplier selection involving four criteria and four alternatives. The criteria weights were obtained using entropy method and have been used to obtain the best alternative using TOPSIS methodology. Since the decision making process is highly subjective sensitivity analysis was used to examine the effect of a change in the weights of the different criteria on the final decision. Perturbing the weights systematically to different settings showed that the same decision was obtained. Consequently, the result of the analysis can be relied upon by the decision maker in choosing the best alternative among the four suppliers.

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