FIRST PRINCIPLE ANALYSIS OF HIGHWAY VERTICAL CURVE DESIGN IN COMPARISON TO HIGHWAY DESIGN SOFTWARE OUTPUT

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Abstract
It is very obvious that the design stage of highway construction plays an important role that cannot be inevitably be over emphasize. The geometric design of highway consists essentially of horizontal and vertical alignment design. This paper focus on the vertical alignment design. Software have made it very easy for design and computation to be carried out with lesser time. A vertical curve aids the smooth transition between two sloped roadways, allowing a vehicle to negotiate the elevation rate change at a gradual rate rather than a sharp cut. The vertical curve aid in providing comfort to driver, alignment aesthetics and drainage purposes. The aim of this write up is to carry out an analysis of the vertical curve design using manual method which we otherwise call ***first principle.*** *The approach is to make use of tabular formation of some necessary parameters like the station, distance from BVC, tangent elevation, offset elevation and curve elevation. Two approaches are carried out, first we compute the tangent elevation and then the curve elevation. The analysis is somehow time consuming but become very easy with the use of excel programming. The first principle analysis will aid a highway design engineer to quickly select an adequate K – Value which is a function of length of vertical curve and the grade angle. However, comparing the manual output to that of software output, one can see its 99.9% the same. This manual method will aid a highway geometric designer to know how the finish ground elevations were obtained using any highway design software.*

Keywords: *Vertical curve, Beginning of vertical curve (BVC), End of vertical curve (EVC), Point of Vertical curve (PVI), Tangent elevation, Curve elevation, vertical offset, grade angle.*

1.0 INTRODUCTION

Vertical curve is amongst the two-transition element in geometric design of highway. A vertical curve aids the smooth transition between two sloped roadways, allowing a vehicle to negotiate the elevation rate change at a gradual rate rather than a sharp cut (Garber and Joel, 2001). The vertical curve aid in providing comfort to driver, alignment aesthetics and drainage purposes (Zhongren and Chiu, 2010).

However, the parabolic curve is

majorly used for vertical alignment design rather than the circular curve, this is because it provides a constant rate of change of slope which contributes to smooth alignment transition. The two type of vertical curve used in the geometric design of highway are crest curve and sag curve.

Holding grade change constant and designing a very long vertical curve result in high construction cost and poor drainage performance. On the other hand, too short a vertical curve may lead to poor alignment and inadequate sight distance. Therefore, the essence of vertical alignment design is to determine the minimum vertical curve length that satisfies sight distance requirement, according to grade change, design speed, and other factors, such as drainage and comfort (Zhongren and Chiu, 2010).

2.0 MATERIAL AND METHOD

The proposed road lies between Obe Street and Sabo Uzebba Road. The road is located in Sabongida Ora in Owan West Local Government Area of Edo State and lies between Latitude 06° 54' 24.18" N to 06° 54' 07.68" N and Longitude 05° 55' 68.38" E to 05° 55' 36.42" E (see Figure2.1). The geological formation is predominantly that of the Benin formation. The terrain is mostly flat with slopes ranging from 0 to –1.7%. The topography of the area is comprised of a plain and gentle sloping terrain. The highest point on the traversed route is at chainage 0+200, having an elevation of about 78.95m; while the lowest point is at chainage 0+850, having an elevation of about 71.86m.

The levelling commenced from a Temporary Bench Mark TBM1 (389274.449mE, 320729.152mN) at the centre of the already existing Obe Street with an elevation of 78.355m. Levelling was carried out both along the route centerline and along the cross section of the major routes at an interval of 25m along the longitudinal direction.

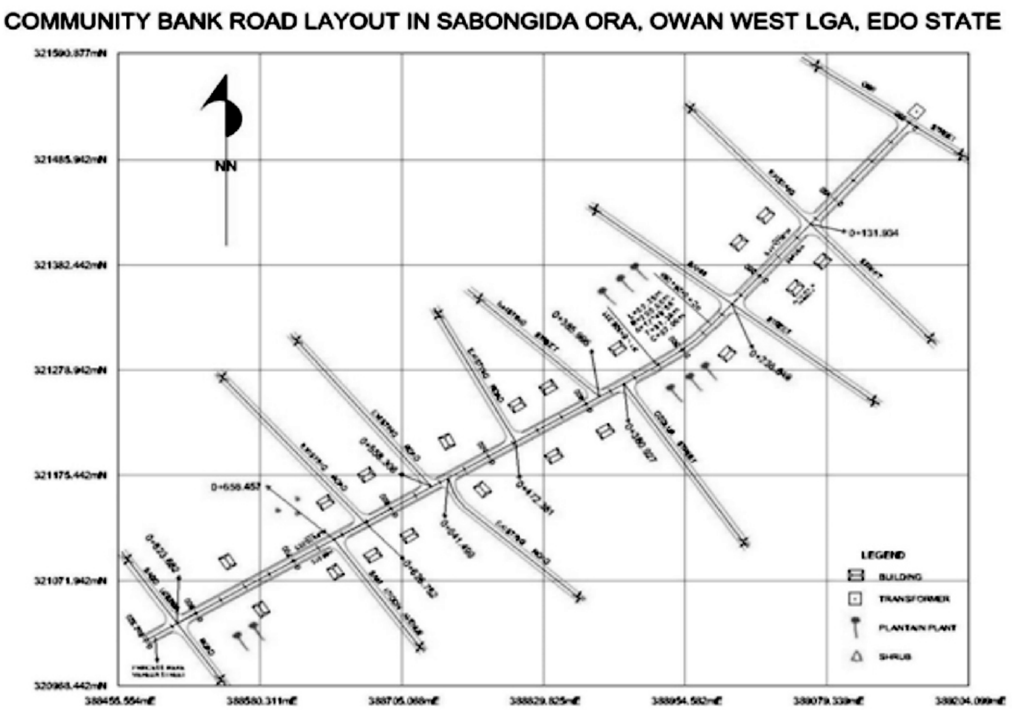

Figure 1: General Layout of Proposed Road

2.1 Design Procedure for Vertical Curves

The design procedure for vertical curves whether crest or sag curve follows the same manner of approach.

1. Determine the minimum length of vertical curve to satisfy sight distance requirements and other criterial for sag curves (comfort, appearance and drainage).
2. Determine from the layout plans the station and elevation of the point where the grades intersect (PVI).
3. Compute the elevations of the beginning of vertical curve, (BVC) and the end of vertical curve (EVC).
4. Compute the offsets, *Y,* (using the equation 2.1) as the distance between the tangent and the curve. Usually equal distances of 100 ft (1 station) are used, beginning with the first whole station after the BVC.

$$Y = \frac{A}{200L}x^2 \quad (1)$$

Where,

Y = tangent offset
A = Algebraic difference of grade, $G_1 - G_2$
x = station distance from BVC

5. Compute elevations on the curve for each station as: elevation of the tangent (±) offset from the tangent, *Y.* Crest curves offset is (–) and for sag curves the offset is (+) (Zhongren and Chiu , 2010).

The figure 2 is a diagrammatic description of a vertical crest curve. The beginning of the curve is the BVC, and the end of the curve is the EVC. The intersection of the grade lines (tangents) is the PVI, which is equidistant from the BVC and EVC.

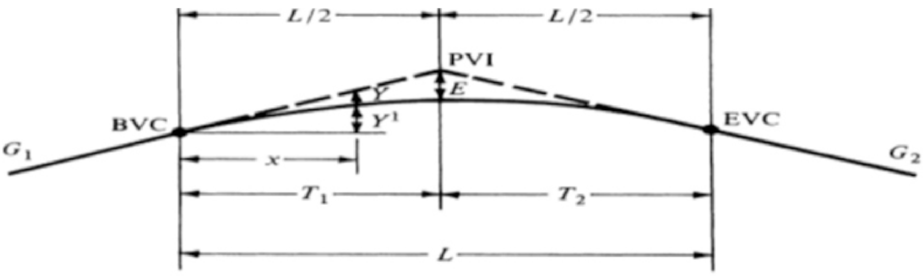

Figure 2: Layout of Crest Vertical Curve

PVI = Point of Vertical Intersection
BVC = Beginning of Vertical Curve
EVC = End of Vertical Curve
E = External Distance
G_1, G_2 = Grade of Tangent (%)
L = Length of Vertical Curve
A = Algebraic Difference of Grades, $G_1 - G_2$

A similar diagram if inverted would apply to a sag vertical curve. Using the properties of a parabola, $Y = ax^2 + bx + c$, where a is constant and b and c is 0, the locations for the minimum and maximum points and the rate of change of slope are determined from the first and second derivative:

$$Y^{1}_{max/min} = 2ax \quad (2)$$

$$\frac{d^2Y}{dx^2} = 2a \quad (3)$$

The horizontal distance between the BVC or EVC and the PVI is $T_1 = T_2 = T$ and the length of the curve L = 2T, where L is in metre. (Recall that the length of the vertical curve is the horizontal projection of the curve and not the length along the curve.) If the total change in slope is A, then

$$2a = \frac{A}{100L} \quad (4)$$

$$a = \frac{A}{200L} \quad (5)$$

The equation of the curve or tangent offset is:

$$Y = \frac{A}{200L}x^2 \quad (6)$$

3.0 RESULT AND DISCUSSION

The software used in the geometric design is Autodesk Land Development. The plan and profile generated by the software is presented below. However, the analysis will be in two approach, along the tangent and along the curve. Subtracting the offset from the tangent elevation gives the curve elevation. The figure 3.1 is a vertical section of the proposed road designed, which happens to be a crest vertical curve with PVI at station 0 + 400.

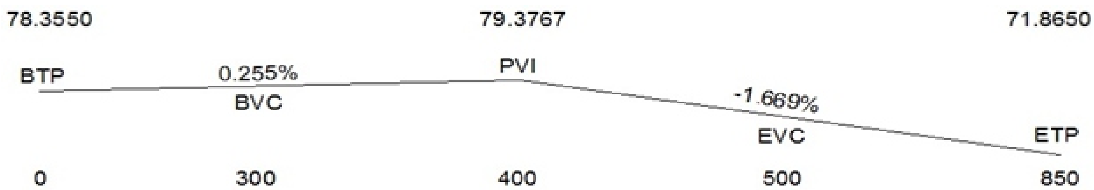

Figure 3: Crest Vertical Curve Section

Where 78.355, 79.377and 71.865 are the respective elevations in metre at the Beginning of Tangent Point (BTP), PVI and End of Tangent Point (ETP) and 0, 300, 400, 500 and 850 are the respective distance at BTP, BVC, PVI, EVC and ETP measured from BTP. The respective slopes are computed below:

$$G_1 = \frac{79.3767 - 78.3550}{400} = \frac{1.0217}{400} = 0.255425\% \approx 0.255\%$$

$$G_2 = \frac{71.8650 - 79.3767}{450} = \frac{-7.5117}{450} = -1.6692667\% \approx 1.669\%$$

The next assignment is to compute the elevations along the tangents at intervals of 25m based on the grade angle and the given elevation at BTP, PVI and ETP.

Let's compute the elevation at station 0+25:

Let L_1 = 25m from BTP
L_2 = 400m at PVI from BTP
$\Delta L = 400 - 25 = 375$
$G_1 = 0.00255425$
$G_1 \times \Delta L = 0.00255425 \times 375 = 0.957844$
H_V = elevation at PVI = 79.3767
$H_T = H_V - (G_1\% \times \Delta L) = 79.3767 - 0.957844 = 78.419$

Let's compute the elevation at station 0+50:

Let L_1 = 25m from BTP
L_2 = 400m at PVI from BTP
$\Delta L = 400 - 50 = 350$
$G_1 = 0.00255425$
$G_1 \times \Delta L = 0.00255425 \times 350 = 0.893988$
H_V = elevation at PVI = 79.3767
$H_T = H_V - (G_1\% \times \Delta L) = 79.3767 - 0.893988 = 78.483$

We can observe that some parameters remain unchanged. The table 3.1 and 3.2 shows the computation of tangent elevation carried out from station 0+000 at BTP to station 0+400 at PVI and to 0+850 at ETP.

Table 3.1: Computation of Tangent Elevation H_V Based on 0.225%

L_2	L_1	$\Delta L = L_2 - L_1$	H_V	Slope	G_1 = (0.255425/100)	. $_1$ x ΔL	$H_T = H_V - (G_1\% \times \Delta L)$
400	0	400	79.3767		0.00255425	1.021700	78.355
400	25	375	79.3767		0.00255425	0.957844	78.419
400	50	350	79.3767		0.00255425	0.893988	78.483
400	75	325	79.3767		0.00255425	0.830131	78.547
400	100	300	79.3767		0.00255425	0.766275	78.610
400	125	275	79.3767		0.00255425	0.702419	78.674
400	150	250	79.3767		0.00255425	0.638563	78.738
400	175	225	79.3767		0.00255425	0.574706	78.802
400	200	200	79.3767	0.255%	0.00255425	0.510850	78.866
400	225	175	79.3767		0.00255425	0.446994	78.930
400	250	150	79.3767		0.00255425	0.383138	78.994
400	275	125	79.3767		0.00255425	0.319281	79.057
400	300	100	79.3767		0.00255425	0.255425	79.121
400	325	75	79.3767		0.00255425	0.191569	79.185
400	350	50	79.3767		0.00255425	0.127713	79.249
400	375	25	79.3767		0.00255425	0.063856	79.313
400	400	0	79.3767		0.00255425	0.000000	79.377

Table 3.2: Computation of Tangent Elevation H_V Based on – 1.669%

L_2	L_1	$\Delta L = L_2 - L_1$	H_V	Slope	$G_1 =$ (1.6692667/100)	G_1 x ΔL	$H_T = H_V - (G_1\% \times \Delta L)$
400	425	25	79.3767	-1.699%	0.016692667	0.417317	78.959
400	450	50	79.3767		0.016692667	0.834633	78.542
400	475	75	79.3767		0.016692667	1.251950	78.125
400	500	100	79.3767		0.016692667	1.669267	77.707
400	525	125	79.3767		0.016692667	2.086583	77.290
400	550	150	79.3767		0.016692667	2.503900	76.873
400	575	175	79.3767		0.016692667	2.921217	76.455
400	600	200	79.3767		0.016692667	3.338533	76.038
400	625	225	79.3767		0.016692667	3.755850	75.621
400	650	250	79.3767		0.016692667	4.173167	75.204
400	675	275	79.3767		0.016692667	4.590483	74.786
400	700	300	79.3767		0.016692667	5.007800	74.369
400	725	325	79.3767		0.016692667	5.425117	73.952
400	750	350	79.3767		0.016692667	5.842433	73.534
400	775	375	79.3767		0.016692667	6.259750	73.117
400	800	400	79.3767		0.016692667	6.677067	72.700
400	825	425	79.3767		0.016692667	7.094383	72.282

From the design carried out, it was earlier stated that the design is carried out in two approaches, the table 3.1 and 3.2 shows the first approach which is that of the tangent elevations. From figure 3.1, it will be observed that the BVC started from station 0+300 and ended at station 0+500, this implies that the computation of the second approach will be from station 0+300 to 0+500. The computation for the second approach is presented in the table 3.3.

Crest Vertical Curve Design Data

Chainage at a PVI = 0 + 400
Elevation at PVI = 79.377
Length of Vertical Curve = 200
Chainage at BVC = 0 + 300
Elevation at BVC = 79.121
Chainage at EVC = 0 + 500
Elevation at EVC = 77.707
Approach Gradient (G_1) = 0.255425%
End Gradient (G_2) = – 1.6692667%
Grade Angle: $A = G_1 - G_2 = 0.255425 - - 1.6692667 = 1.9246917\%$
Absolute Value of $A = (G_1 - G_2) = 1.9246917$
K – Value (L/A) = 200/1.924 = 103.950

Table 3.3: Computation of Crest Vertical Curve

Station	Distance from BVC (x)m	Tangent Elevation (m)	Offset Y = $\left[\frac{A}{200} x^2\right]$m	Curve Elevation = (Tangent Elevation – Offset)m	Remark
0 + 300	0	79.121	0	79.121	BVC
0 + 325	25	79.121+25x0.00255425= 79.185	0.030	79.155	
0 + 350	50	79.121+50x0.00255425= 79.249	0.120	79.129	
0 + 375	75	79.313	0.271	79.042	
0 + 400	100	79.377	0.481	78.896	New Height at PVI
0 + 425	125	79.441	0.752	78.689	
0 + 450	150	79.505	1.083	78.422	
0 + 475	175	79.569	1.474	78.095	
0 + 500	200	79.633	1.925	77.708	EVC

From the table 3.3, it will be observed that the tangent elevation computation was carried out for chainage 0+325 and 0+350, this is because a constant value of **0.06385625** was added successively. Since the chainage is at regular interval of 25m, from the computation of the tangent elevation, the only changing parameter is (x)m from BVC at successive 25m interval.

4.0 CONCLUSION

- The essence of manual computation it to check any highway design software output.
- Though the method can be somehow tedious but can be made easy by making use of the Excel programming tool.
- In addition, this manual method also aids in quick selection of an adequate K – Value which is a function of length of vertical curve and the grade angle in satisfaction to Nigeria geometric deign code.
- However, comparing the manual output to that of software output, one can justify it is 99.9% the same.
- The novelty in this work is to equipped a highway geometric design engineer to know how the finish ground elevations were obtained using any highway design software.

REFERENCES

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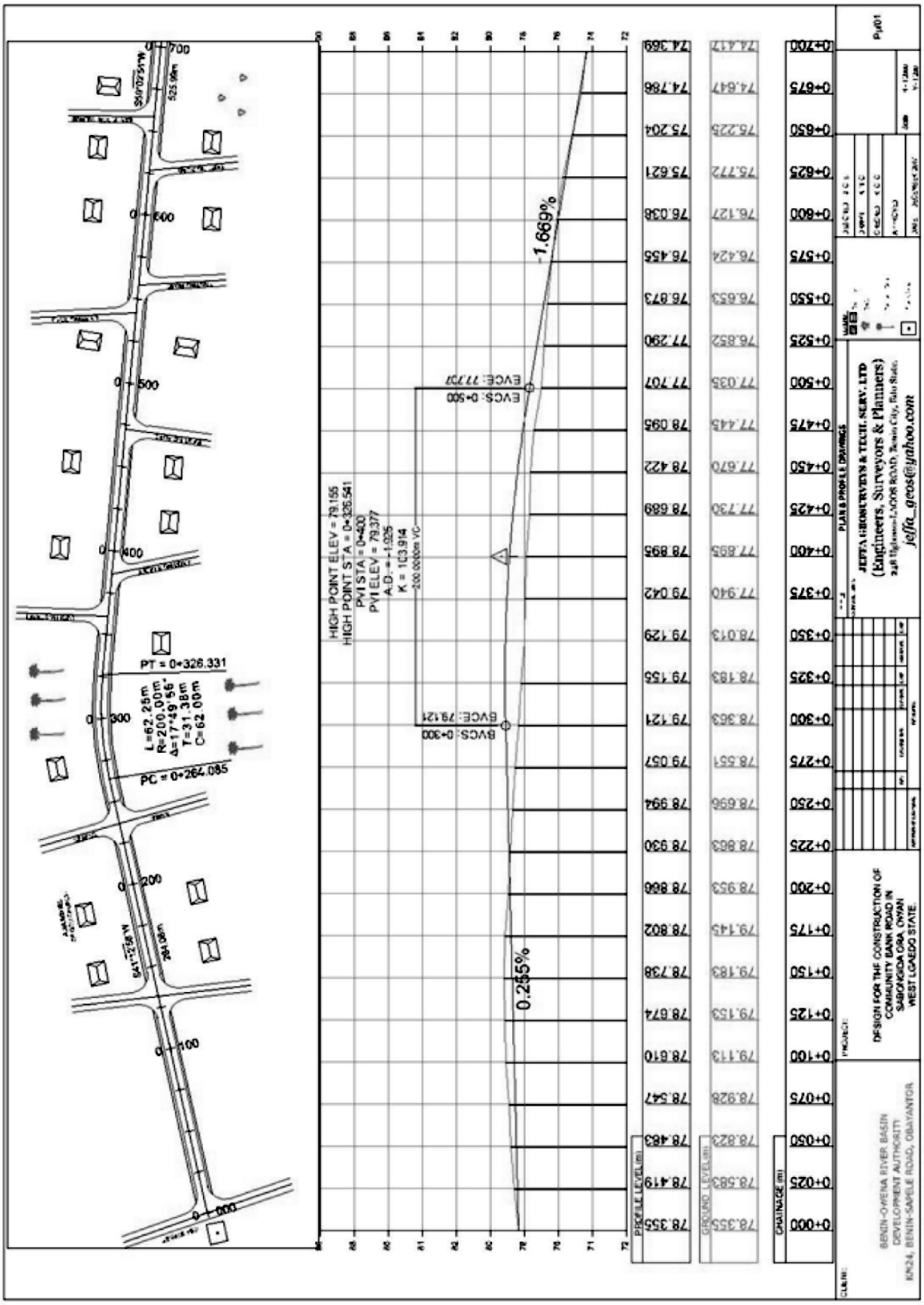
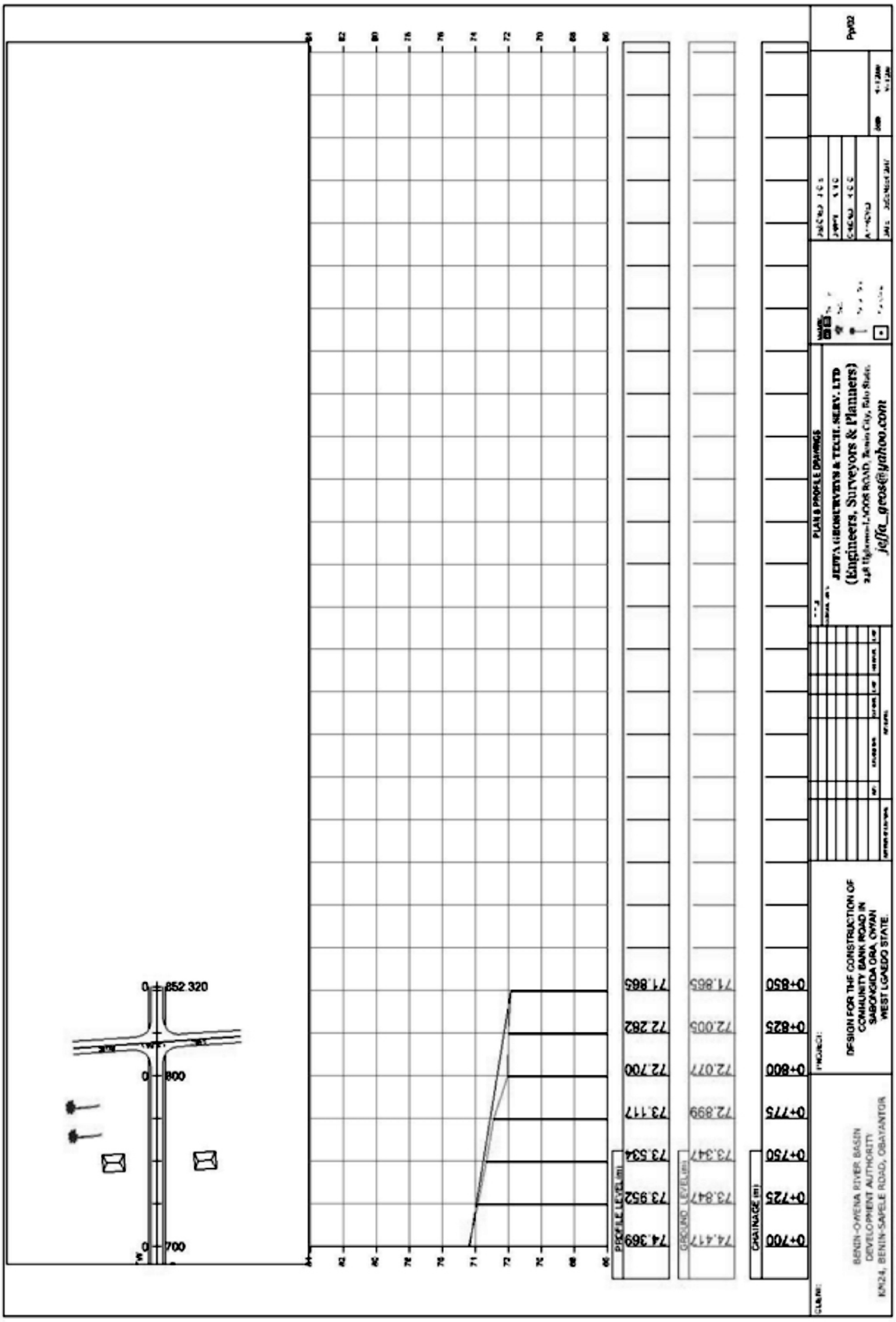
Zhongren W. and Chiu L. (2010). A Critique on the Highway Vertical Curve Design Specifications in China. ICCTP 2010: Integrated Transportation Systems


HIGH POINT ELEV = 79.155
HIGH POINT STA = 0+326.541
PVI STA = 0+400
PVI ELEV = 79.377
A.D. = -1.926
K = 103.914
PT = 0+326.331
PC = 0+264.085
L=62.25m
R=200.00m
T=31.38m
C=62.00m
-1.669%
0.255%
BVCS: 0+300
BVCE: 79.121
EVCS: 0+500
EVCE: 77.707
PROFILE LEVEL (m)
GROUND LEVEL (m)
CHAINAGE (m)
(Engineers, Surveyors & Planners)
jeffa_geos@yahoo.com


PROFILE LEVEL (m)
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GROUND LEVEL (m)
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72.077
72.005
71.865
CHAINAGE (m)
0+700
0+725
0+750
0+775
0+800
0+825
0+850
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DESIGN FOR THE CONSTRUCTION OF COMMUNITY BANK ROAD IN SABONGIDA ORA, OWAN WEST LGA EDO STATE.
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KM24, BENIN-SAPELE ROAD, OBAYANTOR
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